Modified similarity renormalization of Hamiltonians. QED on the light front.

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Abstract

Modified similarity renormalization of Hamiltonians is proposed, that performes by means of flow equations the similarity transformation of Hamiltonian in the particle number space. This enables to renormalize in the energy space the field theoretical Hamiltonian and makes possible to work in a severe trancated Fock space for the renormalized Hamiltonian.

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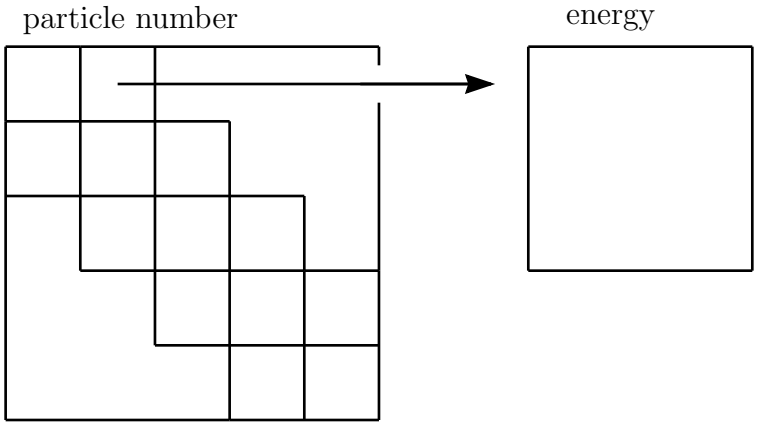

Figure 1: pentadiagonal form of Hamiltonian

1 Motivation

How to solve relativistic bound state problem numerically?

$$H|\psi>=E|\psi> \tag{1}$$

H-canonical Hamiltonian of the field theory.

Problem

H-is infinite dimensional

1.in 'energy' space
(high-energy fluctuations (states) couple to low-energy one)

2.in 'particle number' space (there are interactions in Hamiltonian, which change particle number of a state; therefore in Fock space representation many-body states couple to few-body states)

The methods available to handle

the first problem:
1.Regularization and similarity renormalization of Hamiltonians by Glazek,Wilson
2.Projection of high-energy states onto the low-energy one (Bloch-Fleshbach formalism)

the second problem:
1.Tamm-Dancoff trancation
2.Method of iterrative resolvents by Pauli

The field theoretical Hamiltonian has the 'pentadiagonal' form in the 'particle number' representation, each 'particle number' block in its turn has infinite many states in 'energy' representation.
The way out to construct effective finite dimensional, in 'energy' and 'particle number' space, Hamiltonian,which controls the effects of high-energy and many-body states.

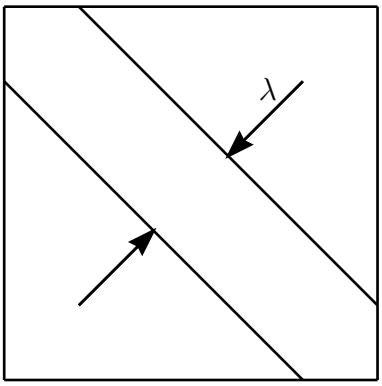

Figure 2: band-diagonal Hamiltonian in 'energy space'

2 Similarity renormalization in 'energy' and 'particle number' space (compared)

I.Idea

'Energy renormalization'	'Particle number renormalization'
Program (Glazek,Wilson)	
1.With the help of similarity transformation decouple	
high-energy	many-body
and low-energy states	and few-body states
2.Simulate the effects of	
high-energy states	many body states

by a set of local operators-counterterms; and new induced interactions, corresponding to marginal relevant operators of the theory

Aim

Effective Hamiltonian H^{eff} is	
band-diagonal in 'energy' space	block-diagonal in 'particle number' space
$\|E_i - E_j\| < \lambda$	each block conserves the number of particles

Choice

1.'Diagonal' part of Hamiltonian H_d is	
H_0-free (noninteracting) part of Hamiltonian	H_c-particle number conserving part of Hamiltonian
2.Basis	
$H_0\|i>= E_i\|i>$	$H_c\|k>= E_k\|k>$
$\|i>$-single particle state	$\|k>$-state with particle number 'k'
Similarity function in 'diagonal' sector is equal to unity	$u_d = 1$

Tool

Flow equations of Wegner	
in 'energy space'	in 'particle number space'

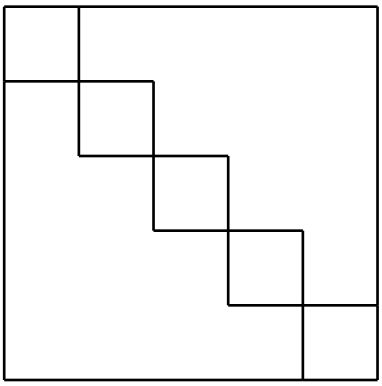

Figure 3: block-diagonal Hamiltonian in 'particle number space'

II.Technique

$$
\begin{aligned}
&H_0, H_c \rightarrow H_d \\
&|i>, |k> \rightarrow |n> \\
&H = H_d + H_r
\end{aligned} \tag{2}
$$

Flow equation (Wegner)

$$
\begin{aligned}
&\frac{dH(l)}{dl} = [\eta(l), H(l)] \\
&\eta = [H_d, H_r]
\end{aligned} \tag{3}
$$

In matrix form

$$
\begin{aligned}
&\frac{dH_{mn}}{dl} = [\eta, H_r]_{mn} - (E_m - E_n)^2 H_{rmn} \\
&\eta_{mn} = (E_m - E_n) H_{rmn}
\end{aligned} \tag{4}
$$

Through the similarity function

$$
\begin{aligned}
&\frac{dH_{mn}}{dl} = [\eta, H_r]_{mn} + \frac{du_{mn}}{dl}\frac{H_{mn}}{u_{mn}} \\
&\eta_{mn} = \frac{1}{E_m - E_n}(-\frac{du_{mn}}{dl}\frac{H_{mn}}{u_{mn}})
\end{aligned} \tag{5}
$$

where the similarity function is

$$
\begin{aligned}
&u_{mn}(l) = \exp(-\Delta_{mn}^2 l) \\
&\Delta_{mn} = E_m - E_n
\end{aligned} \tag{6}
$$

Perturbation theory $H^{(n)} \sim e^n$
'rest sector' (matrix elements with $m \neq n$)

$$
\begin{aligned}
&\frac{dH_r^{(1)}}{dl} = -(E_m - E_n)^2 H_r^{(1)} \\
&H_r^{(1)}(l) = u_{mn}(l) H_r^{(1)}(0) \\
&\eta_{mn}^{(1)} = (E_m - E_n) H_r^{(1)}
\end{aligned} \tag{7}
$$

$$\frac{dH_r^{(2)}}{dl} = [\eta^{(1)}, H_r^{(1)}]_r - (E_m - E_n)^2 H_r^{(2)}$$
$$H_r^{(2)}(l) = u_{mn}(l)\tilde{H}_r^{(2)}(l)$$
$$\tilde{H}_r^{(2)}(l) = \tilde{H}_r^{(2)}(0) + \int_0^l \frac{1}{u_{mn}}[\eta^{(1)}, H_r^{(1)}]_r dl' \quad (8)$$

'diagonal sector' (matrix elements with $m = n$)

$$\frac{dH_d^{(2)}}{dl} = [\eta^{(1)}, H_r^{(1)}]_d$$
$$H_d^{(2)}(l) = H_d^{(2)}(0) + \int_0^l [\eta^{(1)}, H_r^{(1)}]_d dl' \quad (9)$$

1.
'energy renormalization'
(Glazek,Wilson similarity renormalization)
$l = 1/\lambda^2$ (λ-UV-cutoff),
$\forall$ matrix elements holds $|E_i - E_j| < \lambda$
H^{eff} is band-diagonal

'particle number renormalization'
when $l \to \infty$ ($\lambda \to 0$) the 'rest sector' is completly eliminated
H^{eff} is block-diagonal

2.in 'diagonal sector' similarity function is equal to unity $u_d = 1$
3.in the case of QED $H^{(2)}$ simulate (to the leading order) the effects of high-energy (many-body) states.

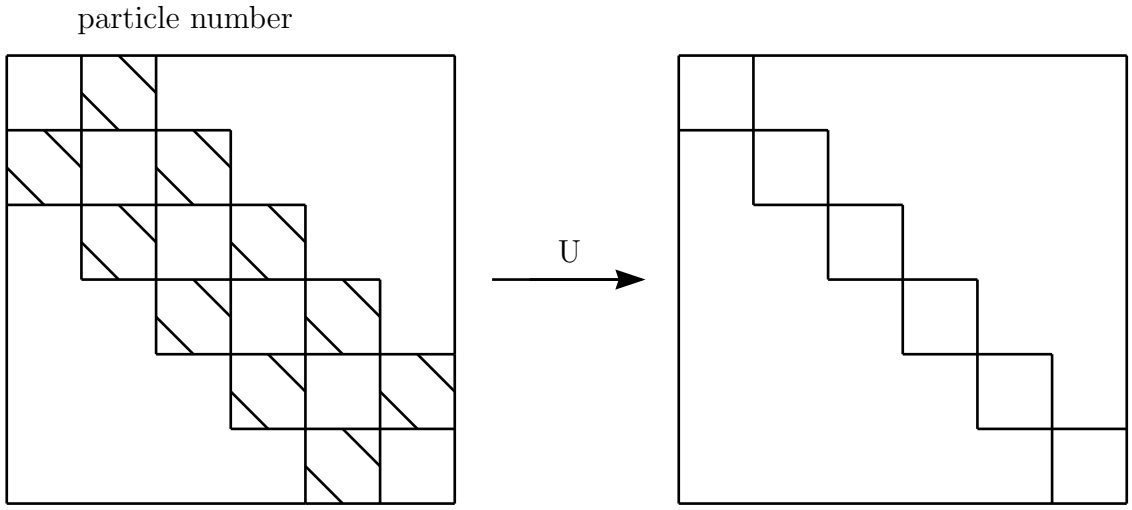

Figure 4: Modified similarity renormalization of Hamiltonians

3 Modified similarity renormalization.

I.Idea
Aim
Perform 'energy' and 'particle number' renormalization simultaneously. Simulate both effects of high-energy and many-body states.
Program
Perform similarity transformation in the 'particle number space' to bring Hamiltonian to a block-diagonal form, with the number of particles conserving in each block.
Tool
Flow equations in the 'energy space', organized in a way to eliminate the particle number changing sectors and to generate effective Hamiltonian in the particle number conserving sectors. The effects of high-energy and many-body states are simulated then by a set of effective interactions, which do not change particle number and do not couple to high-energy states either.
Coice
1.H_d (diagonal part) is the particle number conserving part of the Hamiltonian
that is equivalent to
Similarity function in 'diagonal particle number sector' is equal to unity

$$H_d = H_c \quad \Leftrightarrow \quad u_{dij} = 1 \tag{10}$$

2.Basis

$$H_0|i>= E_i|i> \tag{11}$$

H_0-free (noninteracting) part of Hamiltonian,
$|i>$-single particle state
'Energy' and 'particle number' renormalizations are closely related in a comlicated way.

II.**Technique**

$$H = H_d + H_r$$
$$\eta = [H_d, H_r] \tag{12}$$

H_d-particle number conserving part ('diagonal sector')
H_r-particle number changing part ('rest sector')
Perturbation theory

$$H = H_{0d} + \sum_n (H_d^{(n)} + H_r^{(n)})$$
$$H^{(n)} \sim e^n \tag{13}$$

$$\frac{dH^{(n)}}{dl} = \sum_k [\eta^{(k)}, H_d^{(n-k)} + H_r^{(n-k)}]$$
$$+ \sum_k [[H_d^{(k)}, H_r^{(n-k)}] H_{0d}] + [[H_{0d}, H_r^{(n)}] H_{0d}]$$
$$\eta^{(n)} = [H_{0d}, H_r^{(n)}] + \sum_k [H_d^{(k)}, H_r^{(n-k)}] \tag{14}$$

In matrix form

$$H_{0d}|i> = E_i|i>$$
$$\frac{dH_{ij}^{(n)}}{dl} = \sum_k [\eta^{(k)}, H_d^{(n-k)} + H_r^{(n-k)}]_{ij}$$
$$-(E_i - E_j) \sum_k [H_d^{(k)}, H_r^{(n-k)}]_{ij} - (E_i - E_j)^2 H_{rij}^{(n)} \tag{15}$$

$|i>$-single particle state
'diagonal sector'

$$\frac{dH_{dij}^{(n)}}{dl} = \sum_k [\eta^{(k)}, H_d^{(n-k)} + H_r^{(n-k)}]_{dij}$$
$$+ \sum_k [[H_d^{(k)}, H_r^{(n-k)}]_d, H_{0d}]_{ij} \tag{16}$$

'rest sector'

$$\frac{dH_{rij}^{(n)}}{dl} = \sum_k [\eta^{(k)}, H_d^{(n-k)} + H_r^{(n-k)}]_{rij}$$
$$+ \sum_k [[H_d^{(k)}, H_r^{(n-k)}]_r, H_{0d}]_{ij} - (E_i - E_j)^2 H_{rij}^{(n)} \tag{17}$$

'diagonal sector' new terms are induced;
in 'diagonal sector' the matrix elements with any energy differences are present
'rest sector'
has band-diagonal structure in the 'energy space'

$$H_{rij} = u_{ij} \tilde{H}_{rij}$$
$$u_{ij} = \exp(-(E_i - E_j)^2 l) \tag{18}$$

when $l \to \infty$ the 'rest part' is completly eliminated (exept the diagonal in 'energy space' matrix elements $i = j$, which do not contribute to physical values)
the effective Hamiltonian has block-diagonal form in the 'particle number' space.

4 Bound states in light-front QED (LFQED).

I.Motivation.
QED-perturbation theory in bare coupling constant is applicable;
LFQED-justify the 'parton picture' of bound states as a weakly bound system of the constituents.
II.Positronium on the light-front.
The complete elimination (flow equation in the 'energy space' runs up to the limit value of $l \to \infty$)($\lambda \to 0$)) of the electron-photon vertex ($|ee\gamma>$ sector) gives rise to the new generated electron-positron interaction ($|e\bar{e}>$ sector). The instantaneous interaction (artefact of light-front formulation) stays intact to the leading order by this flow as the particle number conserving interaction.

In exchange channel
$p_1 + p_2 \to p_3 + p_4$,
$p_1 = (x, k_\perp), p_3 = (x', k'_\perp)$-electron

$$V^{gen} = -e^2 M^{ex}_{2ii} \frac{1}{(p_1^+ - p_3^+)} \left(\frac{\int_0^\infty df_{p_1p_3\lambda'}/d\lambda' f_{p_4p_2\lambda'} d\lambda'}{\Delta_{p_1p_3}} + \frac{\int_0^\infty df_{p_4p_2\lambda'}/d\lambda' f_{p_1p_3\lambda'} d\lambda'}{\Delta_{p_4p_2}} \right)$$

$$V^{inst} = -\frac{4e^2}{(p_1^+ - p_3^+)^2} \delta_{s_1s_3}\delta_{s_2s_4} \tag{19}$$

where M^{ex}_{2ii} is the matrix element of two $ee\gamma$ verteces in exchange channel, $M^{ex}_{2ii} = \Gamma(p_1, p_3)\Gamma(-p_4, -p_2)$; $\Delta_{p_1p_3}, \Delta_{p_4p_2}$ are energy denominators, $\Delta_{p_1p_3} = p_1^- - p_3^- - (p_1 - p_3)^-$;

$f_{p_1p_3}, f_{p_4p_2}$ define the velocities of elimination of corresponding $ee\gamma$ verteces in the 'energy space'

$$e_{p_1p_3}(\lambda) = e(\Lambda)\frac{f_{p_1p_3\lambda}}{f_{p_1p_3\Lambda}} \tag{20}$$

$e(\Lambda)$ is the bare coupling constant; $f_{p_1p_3\lambda}$ depends on the transformation performed and is the function of the 'similarity function' $u_{p_1p_3\lambda}$; in the simplest case $f_{p_1p_3\lambda} = u_{p_1p_3\lambda}$ (ansatz of

Wegner $u_{p_1p_3} = \exp(-\frac{\Delta^2_{p_1p_3}}{\lambda^2})$,

Glazek,Wilson $u_{p_1p_3} = \Theta(\lambda - |\Delta_{p_1p_3}|)$).

The electron-positron interaction is (neglecting annihilation channel)

$$V = V^{gen} + V^{inst} \tag{21}$$

Properties of 'V'

1.Generated interaction together with the instantaneous one insure the attractive interaction in the whole parameter range of momenta. To the leading order of nonrelativistic approximation 'V' containes the Coulomb term and spin-dependent interaction, which gives rise to the correct value of singlet-triplet splitting $\frac{7}{6}\alpha^2 Ryd$.
2.'V' is finite in the collinear limit $|x - x'| \to 0$.
3.'State-independence' of counterterms (preliminary). By the numerical calculation the same counterterms must be introduced for the ground and first exited (in quantum number 'n') states to get the cutoff independent physical masses.

5 Advantages of MSR.

I.MSR compared to Tamm-Dancoff trancation (TD) and method of iterrative resolvents (IR).

1.Many-body and few-body states are decoupled in MSR. This enables to work in a restricted Fock (particle number) space and not to encounter the usual difficulties of TD and IR. Namely, the counterterms to be introduced are 'sector-independent' (as compared to TD) and 'state-independent' (as compared to IR).

2.The 'energy renormalization' is performed simultaneously with the 'particle number renormalization'. Therefore all counterterms to the definite order in bare coupling, associated with canonical operators of the theory (relevant and marginal) and also possibly new induced marginal relevant operators, are obtained automatically in the procedure. No additional calculation of perturbation theory corrections is needed.

II.MSR compared to similarity renormalization (SR) of Glazek,Wilson.

1.Restriction of Fock space is no more dangerous. The 'state-independent' counterterms are to be introduced.

2.The artificial cutoff λ (the size of the band $|E_i - E_j| < \lambda$) is not needed.The procedure of MSR is performed to the 'end' (the limit $\lambda \to 0$ ($l \to \infty$) of 'MSR' effective Hamiltonian is well defined), where the 'particle number' changing interactions are completly eliminated.Therefore,

first,no cutoff dependence (except the renormalization group running of couplings) is present in the effective (renormalized) Hamiltonian and hence the physical values;

second,there is no ambiguity in the choice of the step, where the procedure must be stoped (namely in the choice of the final value of λ);

third,no difficulties on the convergency of the perturbation theory occure. Namely,the effective Hamiltonian containes the correct value of the zero'th approximation to calculate the bound states.

3.The elimination of the new induced interactions, corresponding to the marginal relevant operators of the theory, causes the convergency problem of SR. This was observed by Wegner on the example of 1-dimensional problem using flow equations. The convergency of the procedure in MSR is well defined.

References

[1] E. L. Gubanova, F. Wegner, hep-th/9702162, Flow equations in similarity renormalization of QED on the light front.

[2] E. L. Gubanova, F. Wegner, in preparation, Flow equations in modified similarity renormalization of QED on the light front.